GCE AS/A level

0976/01

S16-0976-01

MATHEMATICS – C4
Pure Mathematics

P.M. FRIDAY, 17 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{17 + 4x - x^2}{(2x-1)(x-3)^2}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) **Use your result to part *(a)*** to find an expression for $f'(x)$. [2]

2. *(a)* (i) Expand $\dfrac{1}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^2.

(ii) State the range of values of x for which your expansion is valid. [3]

(b) Use your expansion in part *(a)* to find an approximate value for one root of the equation

$$\frac{6}{\sqrt{1+2x}} = 4 + 15x - x^2.$$ [2]

3. The curve *C* has equation

$$x^4 + 2x^3y - 3y^4 = 16.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{2x^3 + 3x^2y}{6y^3 - x^3}$. [3]

(b) Show that there are only two points on *C* where the gradient of the tangent is -2.
Find the coordinates of each of these two points. [4]

4. *(a)* The angle x is such that $0° \leqslant x \leqslant 180°$, $x \neq 90°$.

Given that x satisfies the equation $3\tan 2x + 16\cot^2 x = 0$,

(i) show that $3\tan^3 x - 8\tan^2 x + 8 = 0$,

(ii) find all possible values of x, giving each answer in degrees, correct to one decimal place. [8]

(b) Express $24\cos\theta - 7\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

Hence, find the range of values of k for which the equation

$$24\cos\theta - 7\sin\theta = k$$

has no solutions. [5]

5. The parametric equations of the curve *C* are

$$x = \frac{3}{t},\ y = 4t.$$

(a) Show that the tangent to *C* at the point *P* with parameter p has equation

$$3y = -4p^2x + 24p.$$ [4]

(b) The tangent to *C* at the point *P* passes through the point (1, 9). Show that *P* can be one of two points. Find the coordinates of each of these two points. [4]

6. *(a)* Find $\int (2x+1)e^{-3x}\,dx$. [4]

(b) Use the substitution $u = 4 + 5\tan x$ to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x}\,dx.$$ [4]

7. The value, £V, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that the initial value of the car is £A, show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

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8. The position vectors of the points *A* and *B* are given by

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$
$$\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k},$$

respectively.

(a) (i) Write down the vector **AB**.

(ii) Find the vector equation of the line *AB*. [3]

(b) The vector equation of the line *L* is given by

$$\mathbf{r} = -\mathbf{i} + 8\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

(i) Given that the lines *AB* and *L* intersect, find the value of p.

(ii) Determine whether or not the line *L* is perpendicular to the vector $6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$, giving a reason for your answer. [7]

9. The region *R* is bounded by the curve $y = \cos x + \sin x$, the x-axis and the lines $x = \frac{\pi}{5}$, $x = \frac{2\pi}{5}$. Find the volume of the solid generated when *R* is rotated through four right angles about the x-axis. Give your answer correct to two decimal places. [6]

10. Prove by contradiction the following proposition.

When x is real and $x \neq 0$,

$$\left|x + \frac{1}{x}\right| \geqslant 2.$$

The first two lines of the proof are given below.

Assume that there is a real value of x such that

$$\left|x + \frac{1}{x}\right| < 2.$$

Then squaring both sides, we have: [3]

END OF PAPER